647510 **BSCmatCC1010** Seat No :________

B.Sc. Semester - 1 (*CBCS*) Examination

Nov./Dec. -2018 (New Course)

MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one of the following questions. **[07]**

(1) State and prove Lagrange's mean value theorem and explain its geometric meaning.

(2) State Tailor's theorem with Lagrange's form of reminder and using that find expansion of $f(x) = e^x$ in terms of x.

Q.1 (B) Answer any one of the following questions. **[04]**

(1) Let a, b ϵ R such that 0 < a < b. Show that there exist c ϵ (a, b) such that $b^b - a^a = c^c[b \log b - a \log a\,]$.

(2) Assuming exactness of expansion expand $f(x) = \frac{1}{2}(sin^{-1}x)^2$ in terms of x.

Q.1 (C) Answer any Three of the following questions. **[03]**

(1)True / False : Every subset of real line R has a least upper bound in R.

(2) Write co-efficient of x^3 in the Maclaurin's series expansion of e^x.

(3) State : Cauchy's mean value theorem.

(4) Give reason why Roll's theorem can not be applied to the function f(x) = |x| , x ϵ [-1, 1] .

(5) Fill the blank. LUB((-1, 1))= ………………

Q. 2 (A) Answer any one of the following questions. **[07]**

(1) (a) Evaluate : $\lim\limits_{x \to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$; (b) Solve : $y^2 + x\frac{dy}{dx} = xy\,\frac{dy}{dx}$.

(2) (a) Evaluate : $\lim\limits_{x \to 1}(1-x)\tan\frac{\pi x}{2}$;

(b) Solve : $(1 + y^2)dx = (tan^{-1}y - x)dy$.

Q.2 (B) Answer any one of the following questions. **[04]**

(1) State and prove necessary and sufficient condition for the differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact.

(2) Solve : $(y^2 - 2xy)\,dx = (x^2 - 2xy)\,dy$.

Q.2 (C) Answer any Three of the following questions. **[03]**

(1) Evaluate : $\lim\limits_{x \to 0}\frac{3^{x^2} - 4^{x^2}}{2x^2}$.

(2) Evaluate : $\lim\limits_{x \to \frac{\pi}{4}}\frac{\log\tan x}{1 - \tan x}$.

(3) State L'Hospital's first rule for $\frac{0}{0}$ form.

(4)Write general form of Bernoulli's differential equation.

(5) Write formula to find general solution of an exact differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$.

Q.3 (A) Answer any one of the following questions. **[07]**

(1) Write general form of differential equation of first order and higher degree solvable for p, explain method to solve it and solve the equation $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$.

(2) Write general form of Lagrange's differential equation, explain method to solve it and solve the equation $y = 2px - (1/3)p^2$.

Q.3 (B) Answer any one of the following questions. **[04]**

(1) Solve : $y = 2px + y^2p^3$.

(2) Solve : $e^{4x}(p - 1) + e^{2y}p^2 = 0$.

Q.3 (C) Answer any Three of the following questions. **[03]**

(1) Write solution of the differential equation $y = p\,x + \frac{m}{p^2}$.

(2) Write general form of Clairaut's differential equation.

(3) True/ False : There exists a Clairaut's form of differential equation which is not in the Lagrange's form.

(4) Solve : $p^2 - 4p + 1 = 0$.

(5) Write an example of a first ordered differential equation having degree 3.

Q.4 (A) Answer any one of the following questions. **[07]**

(1) Let $f(D)\,y = X$ be a linear differential equation with constant coefficient. If $f(D) = (D-a)^r g(D)$ for some $r \epsilon N$, then show that $\frac{1}{f(D)} e^{ax} = \frac{x^r}{r!} \frac{e^{ax}}{g(a)}$.

(2) Solve : $(D^3 - D^2 - 6D)y = (1 + x^2)\, e^x$.

Q.4 (B) Answer any one of the following questions. **[04]**

(1) Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$.

(2) Solve : $(D^2 - 2D + 4)\, y = e^x \cos x$.

Q.4 (C) Answer any Three of the following questions. **[03]**

(1) Write general form of linear differential equation of higher order.

(2) Write corresponding equation of the equation $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - y = x$.

(3) Evaluate : $\frac{1}{D^3 - 4D^2 + 3D - 2}\, e^{-x}$.

(4) Solve : $(D^2 + 1)\, y = 0$.

(5) Explain complementary function.

Q.5 (A) Answer any one of the following questions. **[07]**

(1) Write general form of homogeneous linear differential equation and explain method to convert it in to an equation with constant coefficients.

(2) Solve : $x^2\frac{d^2y}{dx^2} + 4\,x\frac{dy}{dx} + 2y = e^x$.

Q.5 (B) Answer any one of the following questions. **[04]**

(1) Solve : $x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2\,y = x^3 + 3x$.

(2) Solve : $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$.

Q.5 (C) Answer any Three of the following questions. **[03]**

(1) To solve a linear differential equation with variable coefficients, making suitable substitution we convert it in to which type of differential equation?

(2) Find auxiliary equation of $x^2\frac{d^2y}{dx^2} + 7\,x\frac{dy}{dx} + 5y = e^x$.

(3) Find C. F. of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$.

(4) Find particular integral of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$.

(5) Evaluate : $\frac{1}{\theta^2 + 3\theta - 1}\, x^3$.

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